Electrical Engineering
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Tutoring Notes on:

1^{st} Order O.D.E. using "Appropriate Substitution"

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$$\frac{dy}{dx} = (x + y + 1)^2$$

One could try any of the known methods to solve this 1st order ODE, but perhaps the "Appropriate Substitution" method is the fastest [and maybe the only!] way to converge to an answer.

Let: $\boxed{u = x + y + 1}$

Or: $y = u - x - 1$

And: $y' = \frac{d(y)}{dx} = \frac{d(u - x - 1)}{dx} = \frac{d(u)}{dx} - \frac{d(x)}{dx} - \frac{d(1)}{dx} = u' - 1 - 0$

So: $\boxed{y' = u' - 1}$

Therefore: $y' = (x + y + 1)^2 \quad \rightarrow \quad u' - 1 = (u)^2 \quad \rightarrow \quad \boxed{u' = u^2 + 1}$

Now, we could processed as a "Separable Equation": $\frac{du}{dx} = u^2 + 1 \quad \rightarrow \quad \frac{du}{u^2 + 1} = dx \quad \rightarrow \quad \boxed{\int \frac{du}{u^2 + 1} = \int dx + C}$

Use an integration table [or if you still remember the trick from Cal-II CEGEP or MATH-205] obtain the result to the LHS:

From Integration Table: *a>0*

$$\boxed{\int \frac{1}{(u^2 + a^2)} du = \frac{1}{a} \arctan\left(\frac{u}{a}\right)} \quad \therefore \quad \int \frac{du}{u^2 + 1} = \boxed{\arctan(u) = x + C}$$

To get rid of the "arctan" we must take the "tan" on both sides: $\tan(\arctan(u)) = \tan(x + C)$

So: $u = \tan(x + C)$

But, who cares about "*u*" we want our result in "*x*". "*u*" was only used as an intermediate variable to simply our manipulation: $x + y + 1 = \tan(x + C)$

We isolate for "*y*" to get the final answer which is:

$$y = \tan(x + C) - x - 1$$

Let's check if we can get the same result with MATLAB®:

-x-1+tan(x-C1)

Same result!

Except the constant, it shows negative with Matlab and we have positive.
But who cares, its juts a scalar value and the minus could be "absorbed" by the constant!
It is as such based on the convention used by the Matlab programmer of where to put the constant.