Proof by Induction

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Ex #1: prove by induction $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

1° $\boxed{f(n) = \sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}}$ ← $n \in \mathbb{N}^*$

assume this is True

2° Prove that: $\boxed{f(n+1)} = \dfrac{(n+1)(n+2)(2(n+1)+1)}{6}$

$$= \boxed{\frac{(n+1)(n+2)(2n+3)}{6}}$$

3° Proof:

$$f(n+1) = \sum_{i=1}^{n+1} i^2 = \sum_{i=1}^{n} i^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2$$

$$= \frac{n(n+1)(2n+1) + 6(n+1)^2}{6} = \frac{(n+1)}{6}\left[n(2n+1) + 6(n+1)\right]$$

$$= \frac{(n+1)}{6}\left[2n^2 + n + 6n + 6\right]$$

$$= \frac{(n+1)}{6}\underbrace{\left[2n^2 + 7n + 6\right]}$$

$a = 2$
$b = 7$
$c = 6$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-7 \pm \sqrt{49 - (4)(2)(6)}}{(2)(2)}$$

$$= \frac{-7 \pm 1}{4} = \begin{cases} -3/2 \\ -2 \end{cases}$$

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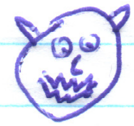
$$\therefore \boxed{f(n+1)} = \frac{(n+1)}{6}(n+2)(n+3/2)2$$

$$= \boxed{\frac{(n+1)(n+2)(2n+3)}{6}}$$

QED ■



EX#2 Prove by induction: $\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\cdots\left(1+\frac{1}{n}\right) = n+1$

1° $\boxed{f(n) = \prod_{i=1}^{n}\left(1+\frac{1}{i}\right) = n+1}$ ← $n \in \mathbb{N}^*$ assume this is true

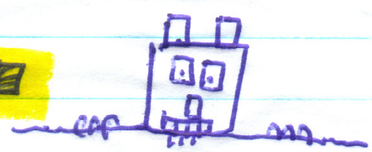
2° Prove that: $\boxed{f(n+1)} = (n+1)+1 = \boxed{n+2}$ ← Same!

3° Proof:

$$f(n+1) = \prod_{i=1}^{n+1}\left(1+\frac{1}{i}\right) = \left[\prod_{i=1}^{n}\left(1+\frac{1}{i}\right)\right]\left[1+\frac{1}{(n+1)}\right]$$

$$= (n+1)\left[1+\frac{1}{(n+1)}\right] = (n+1) + 1 = \boxed{n+2}$$

QED ■



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